

# MATHEMATICAL MODELING OF MAXIMUM POWER OF SOLAR PHOTOVOLTAIC SYSTEMS AND REGRESSION ANALYSIS FOR FIELD TEST CONDITIONS

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## ABSTRACT

Recent Population growth, urbanization and the introduction of numerous electrical appliances have given rise in the demand for electrical energy. The choice of a particular source of energy depends on a number of factors including cost of generation, availability of resources, environmental effects, among other considerations. The aim of this research is to develop a Mathematical model to predict current, Voltage and Power using the diode equivalent circuit and the characteristic obtained from the Datasheet as provided by the manufacturer under the Standard Test Conditions (STC). The photovoltaic of the PV modules type of PS-P310-36 mounted on top of stand situated at Galtimari Primary School Jere Local Government, Borno State, Nigeria was used. Also, Secant numerical method was used in solving resulting nonlinear equations. Likewise, the maximum value of Current, Voltage and Power were obtained and compared which gave errors -3.02%, 6.30% and 3.13% respectively. Furthermore, models that can be used to determine a field test condition were proposed which provide real answer to the user and may help in the design of these systems.

**Keywords:** Photovoltaic, PV Model, Maximum Power Point,  $I - V$  and  $P - V$  curves.

## 1 INTRODUCTION

Solar energy is a renewable free source of energy that is sustainable and totally inexhaustible and is widely used. Many researchers have developed models to predict the potentials of this renewable energy. Abdullahi *et al.* [1] developed a model using Gunn-Bellan radiation integration. This model was used to evaluate the potential of the solar radiation for electricity generation in Potiskum, Yobe State, Nigeria. Luqman *et al.* [2] developed a pre-assessment model using single variable and multi- variable regression techniques of Angstrom type. The model was used to estimate the total solar radiation reaching a horizontal surface in Maiduguri, Borno State, Nigeria. Amusat *et al.* [3] developed Mathematical models to predict monthly average daily global radiation of Maiduguri, Borno State, Nigeria. The results of the models were compared and regression of coefficient was used to determine the model that is best fit for the data. Unlike fossil fuels that are finite. The aggressive consumption rate of fossil fuels has created unacceptable environmental problems such as greenhouse effects, which may lead to disastrous climatic consequences. Thus, renewable and clean energy such as that obtained by using solar cells is required to maintain the quality of human life as well as the environment [4]. The global need for energy savings requires the usage of renewable sources in many applications. One of the renewable sources of energy is the photovoltaic (PV) solar energy. Solar cells in PV is made of semi conducting materials, which absorb sunlight and provide heat energy to freely move the electrons to produce electricity. Solar cells are usually combined into modules which further mounted in PV arrays of several meters in length. Flat-plate PV arrays can be mounted at a fixed angle facing the sun, or can be mounted on a tracking device that follows the sun to capture the most sunlight over the day. The number of PV arrays depends on the requirements of the electricity. For example, house hold applications require 10-20 PV arrays which is enough to provide the power, while large

electricity demands hundreds of arrays which are interconnected to form a single PV system [4]. The evaluation of the maximum power produced by a PV generator is very important to size a PV system. Several models have been developed both experimental and numerical methods to determine the maximum power produced by a PV generator [5]. Zainab and Adel [6] proposed Maximum Power Point Tracking Based on Estimated Power for PV Energy Conversion System. The proposed method depends on the power difference between an estimated maximum power ( $P_e$ ) and the actual power from the PV panel ( $P_{pv}$ ). If the difference is greater than a permitted error value ( $\epsilon$ ), the estimated value for the power will be multiplied by a factor ( $k$ ), and the process will stop when the difference between the estimated and the actual power is equal or less than the error value. The value for  $k$  can be chosen 1 to 2, usually 1.5 [6]. The power error  $E$  will feed a PID controller, which will provide the signal to fire the gate of the boost converter that will increase the voltage to the value required for the maximum power. Guiza *et al.* [7] performed a practical implementation of Perturb and Observe (PO) algorithms based on Arduino microcontroller for tracking of the maximum power generation from PV system under a rapid change in the radiation level. The proposed control system algorithm obtains the data from the PV system through microcontrollers Analog and Digital ports to perform the Pulse Width Modulation (PWM) to the DC-DC converter. A PO Maximum Power Point Tracking (MPPT) algorithm for the PV system was implemented using a DC-DC boost converter and the Arduino microcontroller to control the PWM signal of the converter for achieving MPPT. Heng *et al.* [8] examined a new Newton-based extremum seeking MPPT algorithm for photovoltaic systems. The flow chart was established for the implementation of the proposed Newton-based extremum seeking MPPT method. The controller measures the output voltage and current and then computes the output power. Then the first derivative and second derivative of the power map are estimated, respectively, based on the product of output power and stochastic signals. Reis *et al.* [9] studied mathematical methods to obtain parameters of photovoltaic modules for static and dynamic studies in medium voltage electrical energy systems. These parameters were used for the simulation of this generation system and the reconstruction of I-V and P-V curves of photovoltaic systems in power systems. The voltage and current curves of the photovoltaic module were obtained through the mini KLA meter. The meter operates with a temperature range of -20 to 100°C and an accuracy of  $\pm 0.4\%$ . The simulated curves were able to reach the points of maximum power and that their shape is close to the real curve. Most of the experimental works required field testing which is costly, time-consuming and depends heavily on prevailing weather conditions. Also, adequate security must be provided at the test site. To overcome these problems, this study developed Mathematical model for estimating the maximum electrical power, Current and Voltage that a PV panel generates and also developed Mathematical model to field condition relating to the PV system.

## 2 MATERIALS AND METHOD

### 2.1 Materials

The photovoltaic modules used in this work consist of four mono-crystalline photovoltaic modules type of PS-P310-36 mounted on top of stand situated at Galtimari Primary School Jere Local Government, Borno, Nigeria. The modules were mounted and tilted at an angle  $11.8^\circ$  that is equal to the latitude of the location. Figure 1 shows the photovoltaic mounted on the top of stand. Technical characteristics of the photovoltaic modules used for modeling are as follows: short-circuit current at reference condition  $I_{sc} = 9.31\text{A}$ , open-circuit voltage at reference condition  $V_{oc} = 42.9\text{ V}$ , current at maximum-power point at reference condition  $I_{mp} = 8.54\text{ A}$ , voltage at maximum-power point at reference condition  $V_{mp, ref} = 36.3\text{ V}$ , power at maximum-power point at reference condition  $P_{mp, ref} = 310\text{ W}$ . These are provided by the manufacturer for the reference conditions of  $1000\text{ W/m}^2$  of irradiance level,  $25^\circ\text{C}$  of cell temperature, and 1.5 of air mass (AM). The photovoltaic modules under study are made up of 72 cells grouped in the form of 36 series cell ( $N_{sc} = 36$ ), and 2 parallel cell  $N_{pc} = 2$  absence of modules connected in series ( $N_{sm} = 1$ ) [10].



Figure 1. PV modules of 310 W each, installed in parallel and constituting the photovoltaic panel system installed at the Galtimari Primary School Jere Local Government, Borno State, Nigeria.

## 2.2 Methods

The researchers proposed method is generally based on the use of most vital electrical characteristic of photovoltaic module in figure 2 and manufacturer module datasheet at standard rating condition as an input to obtain cell characteristic values in Table 1 with the aim of developing Mathematical model for the maximum electrical power point (MPPT) that a PV panel generates and acquiring practical outcome through the numerical approaches [11]. One diode photovoltaic module with five parameters whose equivalent diagram is presented in Figure 2.

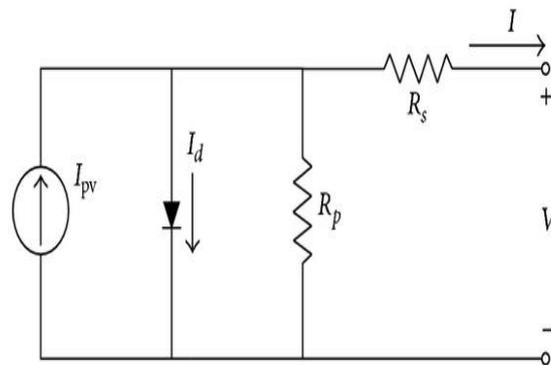


Figure 2: Equivalent Circuit of a Photovoltaic with one diode [12].

The standard equivalent electrical circuit model of PV cell denoted by a single diode in Figure 2 is given by equation (1) [13].

$$I = I_L - I_D - I_p \quad (1)$$

Where  $I_p$  and  $I_D$  are denoted by equations (2) and (3).

$$I_p = \frac{V + IR_s}{R_p} \quad (2)$$

$$I_D = I_o \left( \exp \frac{V + IR_s}{V_t} \right) \quad (3)$$

Substituting equations (2) and (3) in equation (1) gives equation (4).

$$I = I_L - I_o \left( \exp \frac{V + IR_s}{V_t} - 1 \right) - \frac{V + IR_s}{R_p} \quad (4)$$

Where,  $I_p$  - represent the leakage currents, A;  $V_t$  – Thermal Voltage;  $I$ - cell current (A);  $I_L$  – Photo generated current (A);  $I_D$  - Diode reverse saturation current (A);  $q$  - Electron elementary charge ( $1,602 \times 10^{-19}$ coulombs);  $K$ -The Boltzmann constant ( $1,381 \times 10^{-23}$ J/K);  $T$  - Cell temperature (K);  $m$ - Diode ideality factor;  $R_s$ - Cell series resistance ( $\Omega$ );  $R_p$ - Cell shunt resistance ( $\Omega$ );  $V$ - Cell output voltage (V).

### 2.2.1 The Secant Method

The numerical method used in this research was secant method which was evaluated from the Newton-Raphson method of solving a nonlinear equation  $f(x) = 0$  as given by the iterative formula in equation (5).

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \tag{5}$$

With availability of symbolic manipulators of MATLAB, this process has become more convenient. However, it still can be a laborious process, and even intractable if the function is derived as Part of a numerical scheme. To overcome these setbacks, the derivative of the function,  $f(x)$  is approximated as.

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \tag{6}$$

Substituting Equation (6) in equation (5) gives equation (7).

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})} \tag{7}$$

Where  $i$  is the number of the Iteration that is  $i = 1, 2, 3, \dots$  until certain stopping criterion is satisfied (required solution accuracy or maximal number of iterations is reached). The above equation (7) is called the secant method which requires two initial guesses  $x_0$  and  $x_1$ . The absolute relative approximate error  $|\epsilon|$  at the end of iteration is

$$|\epsilon| = \left| \frac{x_i - x_{i-1}}{x_i} \right| \times 100 \tag{8}$$

Where  $i$  is the number of iteration.

## 3 RESULTS AND DISCUSSION

### 3.1 Theoretical results of Series Resistance and Maximum Power of Solar cell

Lorenzo [14] opined that the effect of parallel resistance is negligible, then it can be considered as  $I_p = 0$ . The generated current and short circuit current are the same, meaning  $I_L = I_{sc}$ . Thus, consider  $\left( \exp \frac{V+IR_s}{V_t} \right) > 1$  in any work conditions, and  $\left( \exp \frac{V+IR_s}{V_t} \right) \gg 1$ . Therefore, the current ( $I$ ) from equation can be written as in equation (9).

$$I = I_{sc} \left[ 1 - \left( \exp \frac{V - V_{oc} + I_{sc}R_s}{V_t} \right) \right] \tag{9}$$

In which  $V_{oc}$  is the open circuit tension. Since  $P = IV$ , so the power cell is given by equation (10).

$$P = VI_{sc} \left[ 1 - \left( \exp \frac{V - V_{oc} + I_{sc}R_s}{V_t} \right) \right] \tag{10}$$

The optimum power output parameters of model were determined by deriving the expression for series resistance and maximum power point of the solar cell in equation (11) and (12) and that these outcomes can be broadened to modules by the laws associated to the series and parallel cells. Consider the equation (9), the series resistance of solar photovoltaic cell is given by equation (11).

$$R_s = \frac{-V + V_{oc} + V_t \ln 1 - \left(\frac{I}{I_{sc}}\right)}{I_{sc}} \tag{11}$$

According to [11], equation (10) can be written in term of the maximum power ( $P_{max}$ ) of a photovoltaic cell and maximum voltage ( $V_{max}$ ) in the cell that produces such power is given by equation (12).

$$P_{max} = V_{max} I_{sc} \left[ 1 - \left( \exp \frac{V_{max} - V_{oc} + I_{sc} R_s}{V_t} \right) \right] \tag{12}$$

By differentiating equation ( 12 )  $P_{max}$  with respect to maximum voltage  $V_{max}$  and equating the right hand side to zero i.e.

$$\frac{dP_{max}}{dV_{max}} = 0 \tag{13}$$

Therefore, the maximum voltage can be written as

$$V_{max} = V_{ov} - I_{sc} R_s - V_t \ln \left( 1 + \frac{V_{max}}{V_t} \right) \tag{14}$$

Also equation (9) can be written in term of maximum current as indicated in equation (15)

$$I_{max} = I_{sc} \left[ 1 - \left( \exp \frac{V_{max} - V_{oc} + I_{sc} R_s}{V_t} \right) \right] \tag{15}$$

### 3.2 Practical Result of Open Circuit Voltage and Series Resistance

The value of the thermal voltage of the solar cell, and the generator module when input the values the cell temperature  $T_c = K = 25^\circ C$  or  $298K$ , the Boltzmann constant as  $k = 1.3806505 \times 10^{-23} \text{ J K}^{-1}$ , the electron charge  $e = 1.60205 \times 10^{-19}$  and it is given the diode ideality factor as (the optimum Condition) is given in equation (16).

$$V_t = \frac{mKT_c}{e} = 0.0256955 \tag{16}$$

The series resistance from equation (9) is calculated in (17)

$$R_s = 0.02537376 \tag{17}$$

The values in Table 1 shows the technical specification of the photovoltaic modules provided by the manufacturer such as short-circuit current at reference condition  $I_{sc} = 9.31A$ , open-circuit voltage at reference condition  $V_{oc} = 42.9 \text{ V}$ , current at maximum-power point at reference condition  $I_{mp} = 8.54 \text{ A}$ , voltage at maximum-power point at reference condition  $V_{mp}$ , ref =  $36.3 \text{ V}$ , power at maximum-power point at reference condition  $P_{mp}$ , ref =  $310 \text{ W}$ . These values were used to obtain characteristic values of cell in Table 2.

Table 1. Electrical specification of PS-P310-36 Photovoltaic Modules provided by the manufacturer in operating condition.

Characteristic Value	Cell	Module
Open Circuit Tension ( $V_{oc}$ ), [ V ]	-	42.9
Short Circuit Current ( $I_{sc}$ ), [ A ]	-	9.31
Maximum Rated Tension ( $V_{max}$ ), [ V ]	-	36.3
Maximum Rated Current ( $I_{max}$ ), [ A ]	-	8.54
Maximum Power ( $P_{max}$ ), [ W ]	-	310
Series Resistance ( $R_s$ ), [ $\Omega$ ]	-	-
Thermal Tension ( $V_t$ ), [ V ]	-	-

Table 2. Electrical specification of PS-P310-36 Photovoltaic a cell, module and Photovoltaic generator obtained from the manufacturer on the field test condition.

Characteristic Value	Cell	Module
Open Circuit Tension ( $V_{oc}$ ), [ V ]	1.1920	42.9
Short Circuit Current ( $I_{sc}$ ), [ A ]	4.6600	9.31
Maximum Rated Tension ( $V_{max}$ ), [ V ]	1.0080	36.3
Maximum Rated Current ( $I_{max}$ ), [ A ]	4.2700	8.54
Maximum Power ( $P_{max}$ ), [ W ]	4.3040	310
Series Resistance ( $R_s$ ), [ $\Omega$ ]	0.0254	0.457
Thermal Tension ( $V_t$ ), [ V ]	0.0257	0.9252

Table 3 is the result obtained from equation (14) by using the secant method in which  $f(x)$  is equal to  $f(V_{max})$  and two initial guesses  $x_0 = 0$  and  $x_i = 1$ , while the optimum power occurs at the point  $V_{max} = 0.9776$ .

Table 3. The Result of Secant Method

$i$	$x(i)$	$f(x(i))$	$ (x(i) - x(i - 1)) $
0	0	-1.0718	
1.0000	1.0000	0.0230	1.0000
2.0000	0.9790	0.0015	0.0210
3.0000	0.9776	-0.0000	0.0014

Equations (18) and (19) denoted the characteristic curves of the current and power for the photovoltaic cell that presently exist in the models. These were obtained by computing the respective values of ( $V_{oc}$ ), ( $I_{sc}$ ), ( $R_s$ ) and ( $V_t$ ) shown in Table 2 into equations (9) and (10).

$$I = 4.66 \left[ 1 - \left( \exp \frac{V - 1.071758}{0.0257025} \right) \right] \tag{18}$$

$$P = 4.66V \left[ 1 - \left( \exp \frac{V - 1.071758}{0.0257025} \right) \right] \tag{19}$$

Figure 3 shows the simulated I-V and P-V characteristic curves of the photovoltaic cell that currently exists in the model.

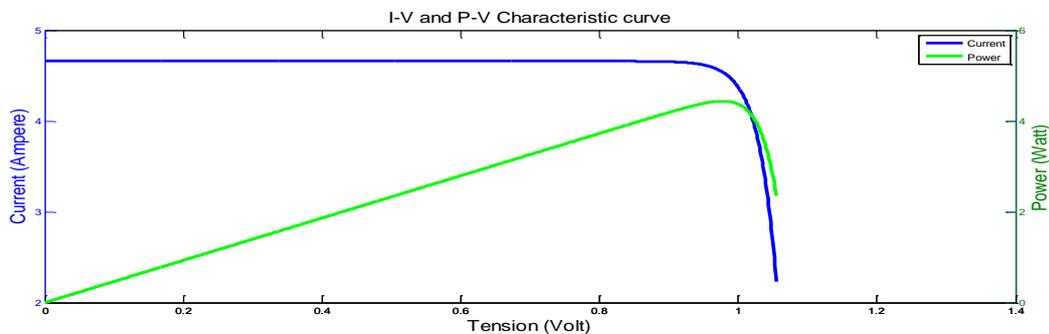


Figure 3. Current and Power curves of the photovoltaic solar cell according to the STC tension condition by the developed equations.

Table 4 shows the distinction between the characteristic value specified by the manufacture and the value estimated using the Mathematical model with their respective error in STC conditions. The maximum power Tension, Maximum power current and Maximum power of photovoltaic module by formulated models are 0.9776, 4.5432 and 4.4388 respectively when compare with that PV module provided by the manufacturer. These errors arose as a result of some electrical phenomena which equation (1) has not been considered which simplified from equation 4 [11].

Table 4. Comparison of characteristic values at point of maximum power of the photovoltaic cell in the STC.

Cell Characteristic Values	Specified Manufacturer	Mathematical Model	Error
Maximum Rated Tension	1.008	0.9776	-3.02%
Maximum Rated Current	4.270	4.5405	6.30%
Maximum Power	4.3040	4.4388	3.13%

The magnitude of variable  $V'$ ,  $I'$  and  $P'$  in the test condition are related as  $V=0.9698 V', I=1.0633 I'$  and  $P = 1.0313P'$  respectively. These values were obtained by comparing the manufacturing specification with the Mathematical Model. Also, these values were inserted into equation (18) and (19) to create new equation for current and power given in equation (20) and (21) respectively [10].

$$I' = 4.38 \left[ 1 - \left( \exp \frac{0.9698 V' - 1.071758}{0.0257025} \right) \right] \tag{20}$$

$$P' = 4.38V' \left[ 1 - \left( \exp \frac{0.9698 V' - 1.071758}{0.0257025} \right) \right] \tag{21}$$

The I-V and P-V curves in Figure 4 show the relationship between the current and voltage output of the photovoltaic cell under standard operating conditions and the adjusted on the Field Test Conditions supply by the manufacturer.

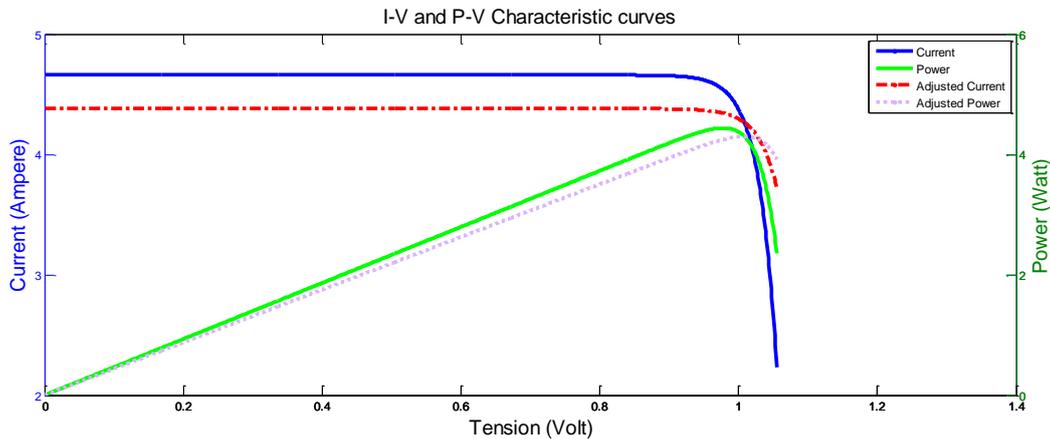


Figure 4. Comparison between the current and power curves of the photovoltaic solar cell and adjusted curves of to the field conditions provided by the manufacturer.

#### 4 CONCLUSION

In this study a Mathematical model of a single diode module type PS-P310-36 was developed. The developed model used manufacturer data to predict the current, voltage and power under STC. The developed model was also used to predict the maximum current, voltage and power. The predicted results were compared with that of manufacturer data and a good agreement was achieved. This model can be applied by the designers of PV systems. It should also be used with caution by considering the manufacturer data covered for the developed model.

## REFERENCES

1. Abdullahi, M., Shodiya, S. and Abdurraheem, A.T. (2018). Solar Radiation Potential for Electricity Generation in Potiskum. ATBU. *Journal of Science, Technology and Education (JOSTE)*, vol. 6, no. 2, pp. 135-141.
2. Luqman, R., Shodiya, S., Ngala, G.M., Oumarou, M.B. and Muhammad A.B. (2016). Pre-Assessment Models of Global Solar Radiation Using Sunshine Duration in Maiduguri. ATBU. *Journal of Science, Technology and Education (JOSTE)*, vol. 4, no. 1, pp. 174-178.
3. Amusat, R.O., Garba, A. A. and Ngadda, Y.H. (2016). Estimation of Global Solar Radiation Using Sunshine –Based Model in Maiduguri, North East, Nigeria. *Applied Research Journal*, vol. 2, no. 1, pp. 19-26.
4. Mandalia, H.C., Jain, V.K., and Pattanaik, B.N. (2012). Application of Super-molecules in Solar Energy Conversion- A Review. *Research Journal of Chemical Sciences*, vol. 2, no. 1, pp. 89-102.
5. Zieba, F.R., Dadjé, A., Djongyang, N. and Doka, S.Y. (2016). New Analytical Modeling Method For Photovoltaic Solar Cells Based On Derivative Power Function. *Journal of Fundamental and Applied Sciences*, vol. 8, no. 2, pp. 426-437.
6. Zainab, A. and Adel, M. (2016). Maximum Power Point Tracking Based on Estimated Power for PV Energy Conversion System. *International Journal of Electrical, Computer, Energetic, Electronic and Communication Engineering* vol. 10, no. 4, pp. 532-537.
7. Guiza, D., Ounnas, D., Soufi, Y. and Maamri, M. (2019). Implementation of Perturb and Observe Based MPPT Algorithm for Photovoltaic System, pp. 1-5.
8. Heng Li, Jun Peng, Weirong Liu, Zhiwu Huang and Kuo-Chi (2014). A Newton-Based Extremum Seeking MPPT Method for Photovoltaic Systems with Stochastic Perturbations. *International Journal of Photoenergy*, pp. 1-13.
9. Reis, L. R.D., Camacho, J.R. and Novacki, D.F. (2017). The Newton Raphson Method in the Extraction of Parameters of PV Modules. *International Conference on Renewable Energies and Power Quality*, vol. 1, no. 15, pp. 634-639.
10. Filho, L.R.A.G., Analysis and Geometric Modeling of the Power Generated by a System Wind Photovoltaic Solar Hybrid., in Doctorate Thesis Faculty of Agronomic Sciences., Paulista State University, Botucatu, pp. 44-57.
11. Luís, R.A.G.F., Daniel, D.S.V.N., Camila, P.C., Odivaldo, J.S. and Fernando, D.C. (2012). Mathematical Analysis of Maximum Power Generated by Photovoltaic Systems and Fitting Curves for Standard Test Conditions. *Engenharia Agricola*, vol. 32, no. 4, pp. 650-662.
12. Jieming Ma, Ka Lok Man, Ting T.O., Nan Zhang, Sheng-Uei Guan and Prudence, W.H.W. (2013). Approximate Single-Diode Photovoltaic Model for Efficient I-V Characteristics Estimation. *The Scientific World Journal*, pp. 1-7.
13. Beddiaf, Z., Izzeddine, S. and Chander, S. (2018). Simulation of Single-Diode Equivalent Model of Polycrystalline Silicon Solar Cells. *International Journal of Materials Science and Application*, vol. 7, no. 1, pp. 8-10.
14. Lorenzo, E. (1994). Solar Electricity: Ingeniery of Photovoltaic Systems. *Madrid: Graphic Arts Galas*, 338p.