

SENSITIVITY ANALYSIS OF A PRODUCTION RATE OPTIMIZATION MODEL USING BLOW FILM MOULDING MACHINE

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ABSTRACT

This study analyses the impact of concentration on the production of a particular product on the total profit (contribution to profit) of Borno Plastic Industry (BOPLAS); a plastic industry in Maiduguri, North eastern Nigeria. This is based on a research carried out to study the allocation problem of the company. Simplex method of linear programming was employed to formulate mathematical model which were solved using Matlab software. Sensitivity analysis using shadow price was used to identify how much variations in the input value for a given variable will impact the results of the developed model. The coefficient of x_3 , polyvinyl chloride (PVC pipe) and the constraint, right hand side of the inequality of the developed model, were increased by 10%. The results show that, the objective values were also increased from 70.5146 to 6.3126 and 84.4986 respectively. The sensitivity analyses revealed that, PVC and wash hand bowls have contributed to the net benefit (profit) of the company by ₦3, 587, 520 and ₦634, 843.1 respectively. It is recommended that the company should emphasise on the practice of operational research techniques for optimum production.

Key Words: Sensitivity analysis, Simplex Method, Shadow price, Optimization

1 INTRODUCTION

To optimize the production rates using the blow film moulding machine, the simplex method algorithm was employed. The simplex method is a mathematical technique used for linear programming, allowing for the maximization or minimization of an objective function while adhering to a set of linear equality and inequality constraints. Recently investigations show that using optimization models to solve industrial decision making is gaining wide acceptance. A research was conducted on an integrated optimization model of production plan in a large steel manufacturing company. The study aimed to develop a model for maximizing total profit while considering various constraints such as production capacity at each subdivision, maintenance costs, and inventory balance. The methodology employed in the research was rooted in operations research, encompassing problem definition, model development, identification of optimal solutions, and sensitivity analysis. To address and solve the optimization problem, the study utilized the Oracle Crystal Ball software. This software likely facilitated the application of probabilistic and simulation techniques, allowing for a more robust analysis of the model under uncertainty [1]. In the context of this research, the simplex method is applied to formulate a model that optimizes production rates by adjusting variables such as machine settings and raw material properties. This algorithm aids in finding the most efficient combination of these variables to enhance productivity while considering the constraints imposed by the manufacturing process [2]. In tandem with the optimization model, a brief overview of sensitivity analysis is crucial. Sensitivity analysis involves assessing how changes in input variables impact the output of a model. In the context of the blow film

moulding optimization model, sensitivity analysis helps identify the influence of variations in machine settings, raw material properties, and environmental conditions on the production rates. This analysis provides valuable insights into the robustness of the optimization model and allows for the identification of critical factors affecting the efficiency of the blow film moulding process [3].

2 METHODOLOGY

2.1 Blow film moulding

The blow film moulding process, a pivotal method in plastic manufacturing, involves a series of intricate steps aimed at producing diverse plastic products. The process begins with the introduction of raw plastic material, often in the form of granules or pellets, into the hopper of the blow film moulding machine. This raw material is then heated until it reaches a molten state, forming a plasticized material ready for shaping. Subsequently, this molten plastic is extruded through a circular die, creating a continuous tube or bubble of plastic. The size of this bubble is controlled by various factors, including the speed of the extrusion process, air pressure, and the dimensions of the die. As the plastic bubble emerges, it is guided through a cooling process to solidify the material. The bubble is then flattened, creating a thin, continuous sheet of plastic film. This film can be further processed and shaped based on the specific requirements of the intended end product. In the context of BOPLAST, the integration of blow film moulding technology has enabled the production of various plastic products, with a notable focus on the fabrication of chairs [4].

2.2 Simplex Method Algorithm

The objective function (Z), is to maximize the contribution to profit. This is mathematically presented as:

$$Z = ax_1 + bx_2 + cx_3 + dx_4 + ex_5 \quad (1)$$

Subject to the following constraints;

$$\text{Blown: } \alpha_b x_1 + \beta_b x_2 + \gamma_b x_3 + \delta_b x_4 + \varepsilon_b x_5 \leq C_I \quad (2)$$

$$\text{Charge: } \alpha_c x_1 + \beta_c x_2 + \gamma_c x_3 + \delta_c x_4 + \varepsilon_c x_5 \leq C_{II} \quad (3)$$

$$\text{Cooling: } \alpha_G x_1 + \beta_G x_2 + \gamma_G x_3 + \delta_G x_4 + \varepsilon_G x_5 \leq C_{III} \quad (4)$$

Where $x_1 x_2 x_3 x_4 x_5 \geq 0$ (*non-negativity constraint*) [5]. The decision variables are identified below, let; x_1 = the proportion of Garden chairs to be produced. x_2 = the proportion of pp woven sack to be produced. x_3 = the proportion of PVC pipe to be produced. x_4 = the proportion of black shopping bag to be produced. x_5 = the proportion of washing hand bowl to be produced. The constraints based on the duration of blowing (B), charge (C), and cooling (G) for each product. Let $\alpha_b, \beta_b, \gamma_b, \delta_b, \varepsilon_b$ represent the respective durations for blowing, $\alpha_c, \beta_c, \gamma_c, \delta_c, \varepsilon_c$ for charge, and $\alpha_G, \beta_G, \gamma_G, \delta_G, \varepsilon_G$ for cooling, therefore equations 1, 2, 3 and 4 can be presented as equation 5, 6, 7 and 8.

$$\text{Blowing: } 4x_1 + 3x_2 + 7x_3 + 2x_4 + 3x_5 \leq C_I \quad (5)$$

Here, C_I represents the maximum available time for the Blowing process.

$$\text{Charge: } 4x_1 + 6x_2 + 9x_3 + 3x_4 + 5x_5 \leq C_{II} \quad (6)$$

Similarly, C_{II} represents the maximum available time for the Charge process.

$$\text{Cooling: } 6x_1 + 6.6x_2 + 11.5x_3 + 5x_4 + 5.7x_5 \leq C_{III} \quad (7)$$

Where C_{III} represents the maximum available time for the Cooling process. The constraints are determined by the durations of the Blowing, Charge, and Cooling processes for each item. The process times, represented by the coefficients in the constraints, are given for each item. The primary goal is to maximize profit while adhering to the time constraints for each process.

2.3 Model Validation

MATLAB's linprog function incorporates state-of-the-art optimization algorithms, including the simplex method, ensuring robust and reliable solutions. The solution vector 'x' satisfies the given linear constraints:

$$4x_1 + 3x_2 + 7x_3 + 2x_4 + 3x_5 = 10 \tag{8}$$

$$4x_1 + 6x_2 + 9x_3 + 3x_4 + 5x_5 = 11 \tag{9}$$

$$6x_1 + 6.6x_2 + 11.5x_3 + 5x_4 + 5.7x_5 = 12 \tag{10}$$

The obtained values for x_1, x_2, x_3, x_4, x_5 optimize the objective function.

$$Z = 6x_1 + 6.6x_2 + 11.5x_3 + 5x_4 + 5.7x_5 = 12 \tag{11}$$

The solution is consistent with the original problem formulation and objective. It aligns with the goal of maximizing or minimizing the specified objective function while meeting the given constraints. It obtains consistent results, indicating the reproducibility and reliability of the solution.

2.4 Sensitivity analysis

In the forthcoming one-month analysis, a comprehensive examination of the contribution to profit for each product was undertaken. Furthermore, a sensitivity analysis was conducted to identify critical constraints within the production process. The products under consideration for this analysis include PP woven sack, PVC pipe, washing hand bowl, black shopping bag (BSB), and garden chair as shown in table 1. To capture the essential details related to production constraints, specific parameters such as the duration of blowing, charge, and cooling were directly obtained from the machine and presented in Table 4 below. These data points were integral to the subsequent sensitivity analysis, shedding light on the factors that significantly influenced the overall profitability of the manufacturing process. This procedure is in agreement with the work of Abba-Aji *et al* [6].

Table 1. Product and recorded machine time during process

Process time (sec)	Garden chairs	PP woven sack	PVC pipe	BSB	Washing hand bowl	Constraint (C)
Blowing	4	3	7	2	3	10
Charge	4	6	9	3	5	11
Cooling	6	6.6	11.5	5	5.7	12

2.5 Optimal monthly Production Quantity Calculations

The company operates 8 hours per day. In a month (24 working days), the total production time is: $\frac{8\text{hours}}{\text{day}} \times \frac{24\text{days}}{\text{month}} = 192\text{hours/month}$. The cycle time required to produce a single PVC Pipe is 30 seconds, while the production of one dozen Washing Hand Bowls takes 20 seconds. Given these cycle times obtained during machine operations, the optimal values derived from the MATLAB script will now be employed in the calculation of the production quantities for PVC Pipes and Washing Hand Bowls.

2.6 New sales values derived from the developed model

The selling prices for the prioritized products determined by the model, i.e., PVC Pipes and Washing Hand Bowls, are calculated below to establish the new sales values.

2.7 Initial Sales Value without the Model

The initial sales values are also computed based on historical data obtained from the company's report, without the model, for PVC Pipes and Washing Hand Bowls. This is done to facilitate a comparison with the new sales values derived from the model, as calculated below.

2.8 Difference in Sales Value (New - Initial)

The calculation below illustrates the difference in sales values between the new values and the initial values, considering the results obtained from the calculated sales values of PVC pipes and Washing Hand Bowls.

2.9 New Total Sales Value of PVC pipes and washing hand bowls

The new total sales value can be obtained by adding the initial sales value to the difference in sales value: New total sales = Initial total sales + Difference in sales.

2.10 Gross Profit

Gross profit is calculated as the difference between revenue (sales) and total costs of production:

3 RESULTS AND DISCUSSION

Upon executing the MATLAB script with the linear programming problem, the following optimal solution and maximum profit were obtained: Optimal Solution: $x_1=0.0001$ (Garden chairs), $x_2=0.0000$ (PP woven sack), $x_3=5.0418$ (PVC Pipe), $x_4=0.0000$ (Black shopping bag), $x_5=2.1989$ (Washing hand bowl). Objective Value: $Z = 70.5146$.

3.1 Sensitivity Analysis Results

Sensitivity Analysis - Increased Coefficient of x_3 by 10%: Original Solution: $x_1=0.0000$ (Garden chairs), $x_2=0.0000$ (PP woven sack), $x_3=5.0418$ (PVC Pipe), $x_4=0.0000$ (Black shopping bag), $x_5=2.1988$ (Washing hand bowl). Objective value = 76.3126. Sensitivity Analysis – When the Right-Hand Side of the first constraint was increased by 10%. The Solution Obtained was: $x_1=0.0000$ (Garden chairs), $x_2=0.0000$ (PP woven sack), $x_3=6.3509$ (PVC Pipe), $x_4=0.0000$ (Black shopping bag), $x_5=2.0111$ (Washing hand bowl). Objective value: = 84.4986.

3.2 Shadow prices using Lagrange Multipliers (Dual Values) results

constraint 1 (λ_B) ≈ -1.4854 : duration of blowing. Constraint 2 (λ_C) ≈ -1.2937 : charge duration. constraint 3 (λ_G) ≈ 1.9166 : duration of cooling.

3.3 Analysis of Results

x_1 (Garden chairs): A very small proportion. x_2 (PP woven sack): No production. x_3 (PVC pipe): A significant proportion. x_4 (Black shopping back) : No production. x_5 (Washing hand bowl) : A moderate proportion. The Optimal solution prioritizes the production of PVC pipe (x_3) and Washing hand bowl (x_5) while minimizing garden chairs (x_1) and excluding PP Woven Sack (x_2) and BSB (x_4). These values represent the proportions of each product to be produced for maximum profitability.

3.4 Sensitivity Analysis - Increased Coefficient of x_3 by 10%

When the coefficient of x_3 is increased by 10%, the sensitivity analysis results reveal significant impacts on the optimization outcomes. Increasing the importance of x_3 in the profit calculation leads to adjustments in the production plan, placing a greater emphasis on x_3 to maximize profit.

3.5 Sensitivity Analysis - Increased Right-Hand Side of the First Constraint by 10%

When the right-hand side of the first constraint is increased by 10%, the sensitivity analysis results show significant changes in the optimization model. The step-by-step guide on the process and a thorough understanding of how these adjustments affect the outcomes are presented in Appendix two. The new optimal solution (X_{new}) constraint and objective value (Z_{new}) constraint show the impact of increasing the blowing duration constraint by 10%. Relaxing the blowing duration constraint allows for adjustments in the production plan, leading to increased profitability. The sensitivity analysis solution places more

emphasis on producing PVC pipe (x_3) and Washing Hand Bowl(x_5), while minimizing Garden chairs (x_3) and excluding PP Woven Sack(x_2) and BSB (x_4).

3.6 Shadow prices using Lagrange Multipliers (Dual Values):

When analysing shadow prices (dual values), it reveals the impact of adjustments in constraint parameters on the overall optimization outcomes. In other words, dual values indicate the sensitivity of the objective function to changes in the constraints. Negative dual values for the blowing and charge constraints suggest that relaxing these constraints could potentially improve the objective function. Conversely, the positive dual value for the cooling constraint implies that increasing the cooling constraint may lead to a better objective. The optimization efforts focused on the production of polyvinyl chloride (PVC) Pipes and Washing Hand Bowls have yielded substantial improvements in profitability when compared to the previous profit statement. Table 2 provides a detailed breakdown of the contribution to profit and the maximum production rate for both the optimized PVC Pipe and Washing Hand Bowl respectively, demonstrating positive impact of the optimization strategies on overall profitability.

Table 2. Contribution to Profit and Maximum Production Rate Comparison for one month

Product	Previous production rate (unit)	Max. Profit (N)	Previous Optimised production rate (unit)	Optimised Profit (N)
PVC pipe	600	1,055,876.80	4,569.60	3,587,520
W.H.B.	12,000	341,607.20	15,716.88	634,843.10

Note: W.H.B for washing hand bowl, max. for maximum

After optimizing the production rates of the products produced in the industry arriving at optimal production rates of PVC and wash hand bowls, a new profit statement was generated as presented in Table 3.

Table 3. New profit statement for the month of October using the model

N		
Revenue (sales);		8,749,603.10
Variable cost:		
Material	1,950,000	
Power	241,990	
Maintenance	202,250	
Overtime	100,000	
Total Variable cost		2,494,240
Fixed cost:		
Labour	1,507,000	
Courier service	6,000	
Communication	20,000	
Transportation	500,000	
Total fixed cost		2,033,000
Total cost		4,527,240
Total contribution:		
PVC pipe	3,587,520	
Washing hand bowl	634,843.10	
Gross profit		4,222,363.10

This detailed analysis shows the impact of optimizing production using the model on both quantity and revenue, resulting in a higher gross profit compared to the initial workflow without the model. The optimal solution derived from the linear programming model prioritizes the production of polyvinyl chloride (PVC) pipes (x_3) and washing hand bowls (x_5), while minimizing the production of garden chairs (x_1) and excluding PP Woven Sack (x_2) and Black Shopping Bag (x_4). This strategic allocation is carefully designed to maximize profitability. In response to the first Sensitivity Analysis, which involves increasing the coefficient of PVC pipes (x_3) in the objective function by 10%, the production plan

undergoes adjustments. This analysis is a methodical examination of how changes in specific variables impact the overall optimization process. The adjustments in the production plan are a direct result of the altered coefficient. The sensitivity analysis aims to understand how variations in the input parameters influence the outcomes. By increasing the importance of PVC pipes (x_3) in the objective function, the model is exploring how this change affects the overall efficiency and profitability of the production process. The revised optimal solution, emerging from this sensitivity analysis, reflects a reevaluation of the system's priorities. It signifies a shift in the model's strategy to achieve better outcomes. This revised solution is not a mere recalculation but a dynamic response to the adjusted coefficient and its implications on the objective function. Crucially, the higher emphasis placed on PVC pipes (x_3) in the revised optimal solution serves a specific purpose: to further maximize profit. This strategic decision reflects a commitment to optimizing the production plan to achieve the primary goal of enhancing overall financial performance. This outcome is in agreement with the work of Abba-Aji *et al* [5]. Similarly, the second Sensitivity Analysis introduces a distinct adjustment strategy. In this case, the focus is on increasing the Right-Hand Side of the First Constraint, specifically the blowing duration constraint, by 10%. This deliberate modification aims to observe how changes in this constraint affect the overall optimization process. The outcome of this sensitivity analysis is noteworthy—the relaxation of the blowing duration constraint. By increasing its Right-Hand Side, the model allows for more flexibility in this aspect of the production plan. This strategic move is undertaken with a clear objective: to facilitate adjustments that will contribute to increased profitability. The relaxed blowing duration constraint prompts a series of adjustments in the production plan. This responsiveness is a key feature of the sensitivity analysis, demonstrating the model's ability to adapt dynamically to variations in input parameters. The aim here is not just to respond to changes but to leverage them for financial advantage. In this newly derived solution, there's a noticeable shift in emphasis. The model, now operating under the relaxed blowing duration constraint, places increased importance on PVC pipes (x_3) and washing hand bowls (x_5). This strategic emphasis reflects a calculated decision to prioritize the production of these specific items for the purpose of maximizing profitability. The analysis involves Shadow Prices, determined through Lagrange Multipliers (Dual Values), these values indicate the impact of a unit increase in a constraint on the objective function. The shadow prices for the blowing and charge constraints are negative. A negative value suggests that relaxing these constraints allowing more flexibility in production could potentially improve the objective function. This implies that there is room to ease the restrictions imposed by these constraints without compromising the overall objective. In contrast, the dual value for the cooling constraint is positive. A positive value indicates that increasing the cooling constraint may lead to a better objective function. This suggests that allocating more resources or effort to meet the cooling constraint could enhance the overall optimization. The shadow prices, with their positive and negative values, provide valuable insights into the importance of each constraint in the optimization process. These guide decision-making by highlighting which constraints have more significant impacts on the objective function. The information derived from shadow prices guides decisions on potential adjustments in the production plan. It informs the model on where to focus efforts—whether to relax certain constraints to improve efficiency or to strengthen others for a better overall outcome. This guidance is crucial for making informed decisions in optimizing the production planning process. In terms of overall implications, the optimal production plan focuses on emphasizing polyvinyl chloride (PVC) pipes and washing hand bowls for maximum profit. The sensitivity analyses contribute valuable insights into the adaptability of the production plan to changes in coefficients and constraints. The shadow prices, determined through Lagrange multipliers, highlight the significance of each constraint, providing guidance for decision-making and potential adjustments. Considering the operational context, BOPLAST industry operates for 8 hours per day, totalling 192 hours in a month (24 working days). The cycle time for producing one polyvinyl chloride (PVC) pipe is 30 seconds, and for one dozen washing hand bowls is 20 seconds. The profit statement for the month of October, utilizing the model, reports revenue (sales) and gross profit of ₦8,749,603.1 and ₦4,222,363.1, respectively. In comparison, the existing workflow without the model produces lower quantities and generates a gross profit of ₦1, 552,760 per month. The adoption of the linear programming model results in a substantial increase in both quantity and revenue, showcasing its effectiveness in maximizing profits. These results are in concord with the findings of Amosun, *et al* [7].

4 CONCLUSIONS

The model successfully allocated production resources, achieving an optimal solution that prioritizes polyvinyl chloride (PVC) pipes and washing hand bowls for maximum profitability. The optimal production plan minimizes garden chairs and excludes PP woven sacks and black shopping bags, aligning with the company's strategic objectives. The new profit statement, derived from the optimized production plan, projects a monthly revenue of ₦8,749,603.1 and a gross profit of ₦4,222,363.1. In comparison, the existing workflow without the model generates lower quantities and a gross profit of ₦1,552,760 per month. The adoption of the linear programming model significantly enhances both quantity and revenue, demonstrating its effectiveness in maximizing profits. This demonstrates a considerable improvement over the initial workflow without the model and justifies why the company needs to emphasize the production of PVC pipes and wash hand bowls with regards to blow film moulding machine. Sensitivity analyses, including increased coefficients and constraints, provided valuable insights into the adaptability of the production plan. Notably, adjustments in coefficients and constraints, particularly for PVC pipe, influenced the production plan and subsequently the objective function. Shadow prices derived from Lagrange multipliers shed light on the critical constraints. Negative dual values for blowing and charge constraints suggest potential flexibility, while the positive dual value for the cooling constraint implies its significance in achieving the optimal objective.

REFERENCES

1. Cucuk, K., Shanella, N. and Wakhid, A. (2021). An integrated optimization model of production plan in a large steel manufacturing company. *Industrial and Production Engineering*, vol. 38, no. 3, pp. 186-196.
2. Ali, K.Y., Ahmed, K.J., Wanzuki, A.M., Mohammed, F.R., Najah, G. and Mo'ath, A. (2021). Simplex and Interior Point Methods for Solving Budgetary Allocation Linear Programming Problem as IR4.0 Strategy. *Journal of Theoretical and Applied Information Technology*, vol. 99, no. 22, pp. 5339-5350.
3. Hakeem, U.R. (2015). Linear Programming Sensitivity Analysis. Applied Operational Research (AOR), Slide Share, a Scribd Company. *Institute of Quality and Technology Management. University of the Punjab*, pp. 1-20.
4. Laurence, W.M. (2017). The Effects of Creep and other Time Related Factors on Plastic and Elastomers. *Chapter 2, a Volume in Plastics Design Library Book, Third Edition. Science Direct*, pp. 41-60.
5. Abba-Aji, M., El-Jummah, A.M. and Ngala, G.M. (2009). Development of a product production rate optimization model. *Continental J. Engineering Sciences*, vol. 4, no. 2, pp. 26-31.
6. Abba-Aji, M.A., Ogwagwu, V. and Musa, B.U. (2013). Simulation Using Sensitivity Analysis of a Product Production Rate Optimization Model of a Plastic Industry. *Open journal of energy efficiency*, vol. 2, pp. 139-142.
7. Amosun, S.T. and Muhammed, M.A. (2021). Optimization of the Production Capacity of a Manufacturing Company as Linear Programming Model. *Science and Technology Research*, vol. 4, no. 3, pp. 119-124.