EVALUATING THE COEFFICIENTS OF A QUADRATIC COST CURVE FOR A THERMAL POWER PLANT WITHOUT FUEL INPUT DATA

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ABSTRACT

Solving economic dispatch problems in a power system requires fuel cost curve coefficients. Therefore, regularly updating these coefficients is crucial for improving the accuracy of economic dispatch solutions. However, the necessary data to calculate these coefficients may not always be available. This paper proposes a method to estimate the quadratic cost coefficients of a thermal power plant when fuel input data is entirely unavailable. The approach involves computing the average heat rate at the plant's minimum operating capacity using the average heat rate at maximum capacity sourced from the U.S. Energy Information Administration and the type of turbine used. Intermediate heat rate values are determined using the finite difference method. The corresponding fuel input for each heat rate and power output level is then calculated to generate the required fuel input versus power output dataset. The cost curve coefficients are estimated using the least squares method. This approach is tested on several Nigerian thermal power plants, and the estimated coefficients are compared with values from recent literature. The results show a maximum error of 27% compared to existing literature. To enhance the accuracy of cost coefficient estimation, future research should explore incorporating real-time operational data, advanced machine learning techniques, or regional calibration factors to further minimize estimation errors.

Key Words: Cost curve parameters, Economic Dispatch Optimization, Least Squares Method, Finite Difference Approach, Thermal Power Generation, Steam Turbine Valve-Point Impact

1 INTRODUCTION

The input-output curve, which represents the relationship between fuel input (measured in British thermal units per hour) and net real power output, is a crucial characteristic of a thermal power plant from an economic operation standpoint. Several other curves related to the plant's economic performance can be derived from this input-output curve. These include the fuel-cost curve, which is obtained by scaling the input-output curve with the unit fuel cost; the heat-rate curve, which plots the ratio of fuel input to power output against power output; and the incremental cost curve, which shows the incremental cost as a function of power output. The incremental fuel cost is determined by differentiating the cost curve concerning power output [1]. The fuel cost function represents the best polynomial fit to the fuel cost curve, and its coefficients, known as fuel cost coefficients, play a critical role in ensuring the accuracy of economic dispatch solutions. Accurately determining or updating these coefficients is essential for improving the precision of economic dispatch calculations. [2] The general mathematical model for fitting a polynomial to a power plant's cost curve is:

$$F(P_g) = a_0 + \sum_{i=1}^{N} a_j P_g^j + r, j = 1, \ldots, N \quad (1)$$

where F(Pg) is the fuel cost function, Pg is the electrical power output in MW, a_j represents the cost coefficients, j denotes the polynomial order (linear, quadratic, cubic, etc.), and r accounts for modeling errors. The coefficient corresponding to the constant term represents fixed costs, including capital investment, loan interest, taxes, labor costs, staff salaries, and other expenses not directly related to fuel consumption. Due to fluctuations in costs and the aging of power units, periodic re-estimation of cost coefficients is necessary to maintain accuracy in economic dispatch calculations. Several classical methods have been proposed in the literature to determine input-output characteristics, such as the quadratic least squares regression method [3]. Additionally, cost curve coefficient estimation has been framed as an optimization problem where estimation errors are minimized. Heuristic approaches such as the particle swarm optimization algorithm [4], genetic algorithm [3, 5], Nelder-Mead local search algorithm [6], and Cuckoo Search algorithm have been employed to refine coefficient estimation by minimizing errors [7]. The primary requirement for estimating cost curve coefficients is a dataset of fuel input and power output pairs, typically obtained from tests on individual thermal generators. However, these datasets may not always be available. This challenge likely motivated Andreas to propose an approximation technique for the heat rate curve, applicable to power plants where data is available only for maximum and minimum operating capacities though it still relies on at least two data points. The method presented in this work aims to estimate quadratic cost curve coefficients when fuel input data is entirely unavailable. The fuel input at maximum and minimum operating capacities is estimated using the average heat rate at full capacity and the type of turbine in use. Intermediate heat rate values are derived using the finite difference method, enabling the generation of the necessary input-output dataset. The least squares estimation method was then applied to compute the cost curve coefficients.

2 METHODOLOGY

2.1 Evaluation of Fixed Costs

Fixed costs include capital investment, interest on borrowed funds, taxes, labor charges, staff salaries, and other ongoing expenses that remain constant regardless of the power plant's load. Determining fixed costs requires a thorough cost analysis of the plant. However, the economic operation of an electrical power system is primarily achieved by reducing variable costs, as plant operators have limited control over fixed costs. Therefore, this study does not account for fixed cost estimation, as it does not impact economic dispatch calculations.

2.2 Calculation of Fuel Expenses

The quantity of fuel required for electricity generation was determined by the generator's efficiency (or heat rate) and the heat content of the fuel used. The heat rate of a generator varies depending on factors such as the type of prime mover, emission control systems, and other operational aspects. Additionally, different fuels have varying heat content. A basic formula to calculate the fuel consumption per kilowatt-hour (kWh) of electricity is [9]: Amount of fuel used per kWh = Heat rate (in British Thermal.

Units (Btu) per kWh) divided by Fuel heat content (in Btu per physical unit) (2)

Amount of fuel used per kWh= $\frac{\mathrm{HR}}{1000}$ cubic feet (3)

1cubic feet of natural gas [10] =1000 Btu (4)

Amount of fuel used = HR Btu per kWh (5)

Amount of fuel used= $HR * P * 10^3$ Btu per hour (6)

The average tested heat rates (HR_{fi}) for various prime movers using natural gas under full load conditions in 2020 are provided in Table 1.

Table 1. Average Tested Heat Rates [11]

Prime mover type	Average tested heat value (HRfi)
Steam generator	1.0347
Gas turbine	11098
Internal combustion	8899
Combine cycle	7.633

The average tested heat rate HR_{fl} is reported at full load conditions *(Pmax)* so that the maximum fuel input (*Fmax*) is;

$$F_{max} = HR_{fl} * P * 10^3 \text{ Btu per hour} \tag{7}$$

The heat rate at minimum generation HR_{min} can be approximated with,

$$HR_{min} = \frac{P_{min}}{P_{max}} HR_{fl} \tag{8}$$

Haven obtained the heat rate at maximum and minimum generation capacities, the one-dimensional, finite difference method [12] is applied to get the heat rate values for other levels of generation. In applying the finite difference method, the interval HR_{min} to HR_{fl} be divided into N-1equal intervals and the step of approximation h, is calculated thus.

$$h = \frac{HR_{fl} - HR_{min}}{N-1} \tag{9}$$

Then using finite differences, the approximate intermediary heat rate values was obtained using [7];

$$HR_i = HR_{i-1} - \frac{HR_{i-1} - HR_{fl}}{h} \tag{10}$$

Expanding equation (10) and rearranging gives

$$HR_i = \frac{A}{c} \tag{11}$$

Where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ -\frac{h-1}{h} & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & -\frac{h-1}{h} & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -\frac{h-1}{h} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$c = \begin{bmatrix} HR_{min} \\ \frac{HR_{fl}}{h} \\ \frac{HR_{fl}}{h} \\ \vdots \\ \frac{HR_{fl}}{h} \\ HR_{fl} \end{bmatrix} \tag{13}$$

The fuel input for each heat rate value was determined using equation.

$$F = HR * P * 10^3 \quad (14)$$

2.3 Consideration of Steam Turbine Power Plants

For steam turbine generators, the input-output curve exhibits non-linearity and non-convexity, particularly in cases where multiple admission valves are present [4]. The presence of these valves leads to non-smooth input-output characteristics, as they open sequentially to allow more steam intake, resulting in oscillations in the curve. To account for this, the fuel input function incorporates a sinusoidal term, as expressed in equation (15).

$$F_i(P_{gi}) = \left[a_{0i} + \sum_{i=1}^{N} a_{ij} P_{gi}^{j} + r_i\right] + \left|e_i \sin(f_i^{(P_{i\,min} - P_i)})\right| \quad (15)$$

Where a_{oi} , a_{ji} are the fuel cost coefficients of the i^{th} unit, e_i and f_i are the fuel cost coefficients of the i^{th} unit valve-point effects [4,13,14]. Therefore, after determining the fuel input and power output pairs using equations (13) and (14), further refinement is performed using equation (15) to incorporate the valve-point effect. If the cost of natural gas is R dollars per million British thermal units (mmBtu), the fuel inputs are converted into monetary fuel costs (CF) following equation (16).

$$CF_i = R * F_i \quad (16)$$

The quadratic polynomial best fitting the fuel cost function is determined using the least square estimation method.

2.4 Least Square Estimation (LSE)

The least square estimation LSE is a method used to perform curve fitting on input-output data. The polynomial of fit can be any order and the major task is to determine the coefficients of the polynomial of fit. The quadratic cost curve, for example, is a second order polynomial so the three coefficients $a_0, a_1$and $a_2$need to be determined. LSE requires three or more data points and is based on minimizing the sum of the squared errors between the data points and those on the polynomial of fit. For the quadratic curve fitting, it is intended to determine $a_0, a_1$and a_2 such that $f(x) = a_0 + a_1 x + a_2 x^2$ Fits the data y_i in the sense of least square. The least square estimates a_0, a_1 and a_2 are those for which the predicted values of the curve minimize the sum of the squared deviations given by;

$$F(a_0, a_1, a_2) = \sum_{i=1}^{N} (a_0 + a_1 x + a_2 x^2 - y_i)^2 \quad (17)$$

Finding the minimizing values $a_0, a_1$and a_2 in equation (17) is to solve the equations resulting from setting,

$$\frac{\partial F}{\partial a_0} = \frac{\partial F}{\partial a_1} = \frac{\partial F}{\partial a_2} = 0 \quad (18)$$

Solving equation (18) implies solving equation (19) to determine $a_0, a_1$and a_2 [15].

$$\begin{bmatrix} N & \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^3 \\ \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 & \sum_{i=1}^{N} x_i^4 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} y_i x_i \\ \sum_{i=1}^{N} y_i x_i^2 \end{bmatrix} \quad (19)$$

The LSE can be implemented in MATLAB environment using *polyfit* command. The usage form of the *polyfit* command is as follows.

$$x = polyfit(P, F, j); \quad (20)$$

Where P is a row vector of power outputs, F is a row vector of actual cost while j is power of polynomial to be fit for P and F. The function returns x which is a row vector of length j containing the polynomial coefficients in descending powers of P.

2.5 Case Study

The study focuses on four Nigerian thermal power plants: Egbin ST2-5 (combined cycle), Delta II-III, Afam IV-V, and Sapele (steam). Their respective power generation limits are: [16]. Egbin: 118 MW ≤ P ≤ 1100 MW, Delta: 10 MW ≤ P ≤ 110 MW, Afam: 24 MW ≤ P ≤ 543 MW, Sapele: 33 MW ≤ P ≤ 223 MW. For this study, the natural gas cost rate from May 2019, which was 5.65 USD per mmBtu [18], is used.

3 RESULTS AND DISCUSSION

Ten power and fuel input sample points were obtained by incrementing power output in nine steps. The fuel input (in dollars per hour) for each gas power plant was calculated using equation (12). In the case of steam power plants, equation (12) was applied first, followed by refinement using equation (15) to account for the valve-point effect. The valve point effect coefficients for Sapele power plant are $e_i = 200$ and $f_i = 0.0042$ [16]. The results of the power output and computed fuel input at each sample point are presented in Table 2. Using least square estimation, quadratic cost curve coefficients were derived and compared with reference values from literature [16]. The percentage error between estimated coefficients and literature values was calculated using equation (21) and is shown in Table 3.

$$Error = \frac{estimated - literature\ value}{literature\ value} \tag{21}$$

Table 2. Computed fuel input and corresponding generation level

Power plant	Egbin		Delta		Afam		Sapele	
Turbine Type	Combine Cycle		Simple Cycle		Simple Cycle		Steam	
Generation level and fuel input	Generation Level (MW)	Fuel cost ($/h)	Generation Level (MW)	Fuel cost ($/h)	Generation Level (MW)	Fuel cost ($/h)	Generation Level (MW)	Fuel cost ($/h)
	118.00	528	10.00	551	24.00	77	33.00	276
	227.10	2198	21.11	2791	71.67	807	54.11	817
	336.20	5010	32.22	6753	119.33	2306	75.22	1643
	445.30	8965	43.33	12438	167.00	4575	96.33	2755
	554.40	14063	54.44	19846	214.67	7615	117.44	4154
	663.60	20303	65.56	28977	262.33	11424	138.56	5838
	772.70	27686	76.67	39831	310.00	16003	159.67	7809
	881.80	36211	87.78	52407	357.67	21352	180.78	10065
	990.90	45879	98.89	66706	405.33	27471	201.89	12608
	1100.00	56690	110.00	82728	453.00	34360	223.00	15437

Table 3. Thermal plants' quadratic cost coefficients

Power Plant	Cost Coefficients [Proposed method]				Cost Coefficients in literature [16]		Generation limits [17]	
	LinearCoefficients (a_1)		Quadratic Coefficients (a_2)		Linear Coefficients (a_1)	Quadratic Coefficients (a_2)	P_G^{MIN}	P_G^{MAX}
	(a_1)	Error (%)	(a_2)	Error (%)				
Egbin	0.0234	17.61	2.93	25.26	0.0284	3.92	118	1100
Afam	0.0264	8.65	1.48	27.09	0.0289	2.03	24	453
Delta	0.0291	10.74	7.17	10.82	0.0326	6.47	10	110
Sapele	0.0177	21.68	6.82	15.80	0.0226	8.10	33	223

4 CONCLUSIONS

This study presents a straightforward method for estimating quadratic cost curve coefficients for thermal power plants when direct fuel input data is unavailable. The method estimates fuel input at different power output levels based on fuel heat content and power plant type, generating input-output data accordingly. The least square estimation method is then applied to determine fuel cost coefficients. The methodology was tested on selected Nigerian thermal power plants, and the derived coefficients were compared with those reported in recent literature. The results show a maximum deviation of 27%, demonstrating that the proposed method provides a reliable means of estimating cost curves for economic dispatch applications.

REFERENCES

1. Arunesh, D. and Ashish, D. (2013). Classical Approach to Solve Economic Load Dispatch Problem of Thermal Generating Unit in MATLAB Programming. *International Journal of Engineering Research & Technology*, vol. 2, no. 10, pp. 1384-1389.

2. Yusuf, S. (2013). Estimation of Fuel Cost Curve Parameters for Thermal Power Plants Using the ABC Algorithm Turkish. *Journal of Electrical Engineering &Computer Sciences*, vol. 21, pp. 1827-1841.

3. Dedy, K.S. and Rina, A. (2017). Analysis of Input-Output Characteristic and Generation Cost Optimization Using Quadratic Least Square Regression and Dynamic Genetic Algorithm Method: Case StudyPT.PLN PJB UP Gresik. American Institute of Physics Conference Proceedings1818, 020075-020075; Doi: 10.1063/1.4979942.

4. Kel-Naggar, K.M., AlRashidi, M.R. and Al-Othman, A.K. (2009). Estimating the input-output parameters of thermal power plants using PSO. *Energy Conversion and Management*, vol. 50, no. 7, pp. 1767-1772.

5. Al-Khandhar, A.M. and EL-Naggar (2006). A genetic based algorithm for optimal estimation of Input-output curve parameters of thermal power plants. *Electrical Engineering*, vol. 89, pp. 585-590.

6. Gopala Krishna Rao, C.V., Yesuratnam, G., Ravindranath, G. and Sirisha, G. (2018). Estimation of Fuel Cost Parameters for Thermal Power Plants Using LSE-Nelder-MEAD Local Search (Lse-Nm) Optimization. *National Conference Proceeding; National Conference on Trends in Science, Engineering & Technology by Matrusri Engineering College &International Journal of Creative Research Thoughts (IJCRT); IJCRT NTSET*, pp. 140-145.

7. Abanihi, V.K. and Ovabor, K.O. (2019). Economic load dispatch of Nigeria integrated high voltage generation and transmission grid using BAT algorithm. *Nigerian Journal of Technology*, vol. 38, no. 3, pp. 497-504,

8. Voltage Generation and Transmission Grid Using Bat Algorithm. *Nigerian Journal of Technology*, vol. 38, no. 3, pp. 680-687.

9. U.S. Department of Energy (2019). *Energy Information Administration website: Cubic feet of natural gas to British thermal units*. U.S. Department of Energy. https://www.eia.gov/tools/faqs/faq.php?id=667&t=3.

10. U.S. Department of Energy, Energy Information Administration. (2019). *Average tested heat rates by prime mover and energy source, 2007–2017*. https://www.eia.gov/electricity/annual/html/epa_08_02.html.

11. Lakshmi, R. and Muthuselvi, M. (2013). Numerical Solution for Boundary Value Problem Using Finite Difference Method. *International Journal of Innovative Research in Science, Engineering and Technology*, vol. 2, issue 10, pp. 5305-5313.

12. Ramesh, K.Y. and Madhav, S.K. (2018). Solving economic load dispatch problem with valve point effect. *International Research Journal of Engineering and Technology*, vol. 5, no. 11, pp. 1481-1485.

13. Awodiji, O.O. and Komla, A.F. (2015). Economic load dispatch of power system using genetic algorithm with valve point effect. *Springer International Publishing Switzerland*, vol. 1, no. 1, pp. 276-284.

14. Temitope, A., Ayodele, S.O. and Mufutau, A. (2013). Computational Solution to Economic Operation of Power Plants. *Electrical and Electronic Engineering*, vol. 3, no. 6, pp. 139-148.

15. Ndunagu, J.N., Abanihi, V.K. and Ikheloa, S. (2019). Economic Load Dispatch of Nigerian Power NetworkUsing a Hybrid of Evolutionary Programming and Efficient Particle Swarm Optimization. *FUW Trends in Science & Technology Journal*, vol. 4, no. 3, pp. 697-702.

17. U.S. Department of Energy (2019). Energy Information Administration https://www.eia.gov/tools/faqs/faq.php?id=667&t=3ac.

18. U.S. Department of Energy, Energy Information Administration (2019). *Henry Hub natural gas (dollars per million Btu)*. https://www.eia.gov/dnav/ng/hist/rngwhhdm htm.

19. Haruna, Y.S., Yisah, Y.A., Bakare, G.A., Haruna, M.S. and Oodo, S.O. (2017). Optimal Economic Load Dispatch of the Nigerian Thermal Power Stations Using Particle Swarm Optimization. *The International Journal of Engineering and Science*, vol. 6, issue 1, pp. 17-23.